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# Some Words about Myself

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On April 30, 2000, I turned 95. Papers about my life were published in *Uspekhi Matematicheskikh Nauk* on account of my 50th, 60th, 70th, 80th, 85th, and 90th anniversaries.

In this paper, I mainly concentrate on my activities for the last 5–10 years and only occasionally revert to the past.

I begin with my scientific work. In recent years, I have been dealing with approximations on manifolds in the Euclidean space $\mathbb{R}^n$.

A manifold $\Gamma \in \mathbb{R}^n$ can be represented by a smooth surface, or its part, or by a closed domain in $\mathbb{R}^n$. In other words, $\Gamma$ is a closed bounded set. The $m$-dimensional manifold $\Gamma$, $1 \le m \le n$, is covered by a finite number of pieces $\sigma$ each of which is projected onto a corresponding $m$-dimensional coordinate subspace $\mathbb{R}^m \subset \mathbb{R}^n$. This allows us to describe the piece $\sigma$ by functions defined on a certain domain $\Omega \subset \mathbb{R}^m$. These functions are said to locally describe $\Gamma$. In general, they are $k$-times continuously differentiable; in this case, we say that $\Gamma$ belongs to the class $S^k$ and write $\Gamma \in S^k$, $k = 1, 2, \dots$. The boundary of $\Gamma$, which exists if $\Gamma$ is a piece of a smooth surface, is also described by functions that may be of very general character.

On the manifold $\Gamma$ (which is $m$-dimensional in the general case), define a function $f$ from the class $H_p^r(\Gamma)$, where $r > 0$ and $1 \le p \le \infty$.

For the functions $f$ defined on a domain $\Omega \subset \mathbb{R}^m$, introduce the $L_p$-norm

$$\|f\|_{L_p(\Omega)} = \left(\int_\Omega |f(x)|^p dx\right)^{1/p}, \qquad 1 \le p < \infty,$$
$$\|f\|_{L_\infty(\Omega)} = \sup_{x\in\Omega} \operatorname{vrai} |f(x)|,$$

and the $H_p^r$-norm

$$\|f\|_{H_p^r(\Omega)} = \|f\|_{L_p(\Omega)} + \sup_{h\in\mathbb{R}^n} \frac{\|\Delta_h^k f\|_{L_p(\Omega_h)}}{|h|^r}, \qquad r > 0,$$

where $\Delta_h^k f(x)$ is a difference of order $k$ with step $h \in \mathbb{R}^n$ at point $x \in \mathbb{R}^n$ and $\Omega_h$ is the set of points $x \in \Omega$ that are situated at a distance at least $k|h|$ from $\partial\Omega$.

Using these norms (in $\mathbb{R}^m$, $1 \le m < n$), we introduce the norms

$$\|f\|_{L_p(\Gamma)} \qquad \text{and} \qquad \|f\|_{H_p^r(\Gamma)}$$

for the manifold $\Gamma$. Omitting the details, we just note that the norm $\|f\|_{H_p^r(\Gamma)}$ is obtained as a sum of norms $\|f\|_{H_p^r(\sigma)}$ calculated on the pieces $\sigma$ that cover $\Gamma$.

The functions $f$ from the class $H_p^r(\Gamma)$ are approximated on the manifold $\Gamma$ by classical polynomials $\Lambda_N(x)$, $N = 1, 2, \dots$. The notation $\Lambda_N(x)$ combines three types of approximating functions:

$P_N(x)$, algebraic polynomials of degree $N$;

$T_N(x)$, trigonometric polynomials of degree $N$;

$G_N(x)$, entire functions of exponential type $N$.

As the measure of approximation, we take the norm

$$\|f - \Lambda_N\|_{L_p(\Gamma)}, \qquad N = 0, 1, 2, \dots .$$

Among other results, we point out the following theorem.

**Theorem 1.** *A function $f$ belongs to the class $H_p^r(\Gamma)$ ($f \in H_p^r(\Gamma)$) if and only if it can be approximated on $\Gamma$ by functions $\Lambda_N$ with the estimates*

$$\|f - \Lambda_N\|_{L_p(\Gamma)} \le cN^{-r}, \qquad N = 1, 2, \dots, \tag{1}$$

$$\|\Lambda_N\|_{H_p^r(\Gamma)} \le c, \tag{2}$$

*where $c > 0$ is independent of $\Lambda_N$.*

Note that the classical approximation theorem dating back to Jackson, Bernstein, de la Valleé-Poussin, and Zygmund for the class $H_p^r *$ of periodic functions approximated by trigonometric polynomials ($\Lambda_N = T_N$) states that the membership of $f$ in this class is equivalent to its approximability by trigonometric polynomials $T_N$ with estimates (1) and

$$\|\Lambda_N\|_{L_p(\Gamma)} \le c. \tag{2$'$}$$

Property (2′) is weaker than property (2). However, in the present (periodic) case, the following equivalence holds:

$$\{(1), (2)\} \quad \Leftrightarrow \quad \{(1), (2')\} \quad \Leftrightarrow \quad f \in H_p^r(\Gamma). \tag{3}$$

There are other situations when this equivalence holds. For example, when $\Gamma = \mathbb{R}^n$, $\Lambda_N = G_N$ and, in addition, when $\Gamma = \sigma$ is a unit sphere in $\mathbb{R}^n$ and $\Lambda_N = P_N$. However, for arbitrary $\Lambda_N$ and the $\Gamma$ given above, there is no such equivalence.

Thus, unlike properties (1) and (2′), properties (1) and (2) are universal and are equivalent to the membership of the function $f$ in the class $H_p^r(\Gamma)$.

In other works, I established that this equivalence can also be achieved in the norms that are stronger than $\|\cdot\|_{L_p(\Gamma)}$. For example, for the functions $f(x)$ defined on the whole space $\mathbb{R}^n$, we can define the norm

$$|||f|||_{L_p(\Gamma)} = \sup_{y \in \mathbb{R}^n} \|f(x + y)\|_{L_p(\Gamma)}$$

in which properties (1) and (2′) are equivalent to the fact that $f \in H_p^r(\Gamma)$; i.e., property (3) holds.

Some of my works of this series are devoted to the study of manifolds $\Gamma$ that satisfy the classical equivalence (3).

The manifolds $\Gamma$ defined parametrically by trigonometric polynomials

$$x_k = \tau_k(\theta_1, \dots, \theta_m), \qquad k = 1, \dots, n,$$

have been studied from this point of view. In these equalities (which define $\Gamma$), the functions $\tau_k$, defined by trigonometric polynomials, satisfy natural constraints. I found that equivalence (3) holds for such $\Gamma$ in the metric of $H_\infty$ or $C$ (in the metric of continuous functions). In the metric of $L_p$, $1 \le p < \infty$, this equivalence holds in any case for surfaces $\Gamma$ that are algebraically homeomorphic to a sphere, as well as for tori.

My last results, which also include unpublished material, generalize the classical statement that the trace of a polynomial of degree $N$ on a unit sphere $\sigma \in \mathbb{R}^n$ (i.e., a spherical function of degree $N$) represents at the same time a trace of a certain harmonic polynomial of the same degree $N$.

I consider a linear differential operator of degree $2l$, $l = 1, 2, \dots$, in $\mathbb{R}^n$ with constant coefficients that is elliptic and self-adjoint. I also consider an algebraic surface $\sigma = \sigma_s$, $s = 1, 2, \dots$, of degree $2s$, subject to the ellipticity condition. In particular, $\sigma_1$ is a classical ellipsoid in $\mathbb{R}^n$. Due to this condition, $\sigma$ is a boundary of a certain bounded domain $\Omega \subset \mathbb{R}^n$ $(\partial\Omega = \sigma)$.

Consider the following boundary value problem of the first kind:

$$\begin{gathered} LU = 0 \qquad \text{on } \Omega, \\ U^{(\alpha)}|_\sigma = P^{(\alpha)}|_\sigma, \qquad |\alpha| \le l-1, \\ U^{(\alpha)} = \frac{\partial^{|\alpha|} U}{\partial x_1^{\alpha_1} \dots \partial x_n^{\alpha_n}}, \qquad \alpha = (\alpha_1, \dots, \alpha_n), \qquad |\alpha| = \sum_1^n \alpha_j, \end{gathered} \tag{4}$$

where $P = P_N$ is an arbitrary polynomial of degree $N$; solution $U$ to the differential equation must have the same boundary values on $\sigma$ as $P$ has.

I prove the following theorem.

**Theorem 2.** *When $s = 1$, a solution to boundary value problem* (4) *is a polynomial of degree $N$ $(U = U_N)$ for any $P = P_N$.*

When $s > 1$, there exist many polynomials $P = P_N$ such that solution $U$ to problem (4) is not a polynomial of degree $N$ $(U \ne U_N)$.

To prove this theorem, we apply the variational method. These studies are related to certain problems of algebraic geometry; I acknowledge useful recommendations of I.R. Shafarevich.

The results below are obtained as the corollaries to Theorem 2 for $s = 1$, i.e., when $\sigma = \sigma_1$ is an ellipsoid.

We can consider the following general boundary value problem:

$$\begin{gathered} LU = 0 \qquad \text{on } \Omega, \\ U^{(\alpha)}|_\Gamma = f^{(\alpha)}|_\Gamma, \qquad |\alpha| \le l-1, \end{gathered} \tag{5}$$

where $f$ is a function defined on $\Gamma$ and belonging, for example, to the class $H_p^r(\Gamma)$.

We can expand $f$ in series in terms of polynomials,

$$f = P_0 + P_1 + P_2 + \dots,$$

and obtain a solution to the boundary value problem (4) in the form of the series

$$U = U_0 + U_1 + U_2 + \dots,$$

where $U_N$ are also polynomials of degree $N$ that solve appropriate problems (4). Polynomials $P_N$ are estimated by Theorem 1; then, polynomials $U_N$ are estimated on the basis of the estimates known in the theory of variational methods.

Returning to Theorem 1, we note that, when proving its direct part, we used the following considerations.

A function $f \in H_p^r(\Gamma)$ is continued beyond $\Gamma$ on $\mathbb{R}^n$ so that the continued function $f$ belongs to the class $H_p^{r+1/p}(\mathbb{R}^n)$; in this case, the following inequality holds:

$$\|f\|_{H_p^{r+1/p}(\mathbb{R}^n)} \le c\|f\|_{H_p^r(\Gamma)}. \tag{6}$$

Now, the functions $f$ continued on $\mathbb{R}^n$ are approximated by functions $G_N(x)$ of exponential type; then, the corresponding estimates are recalculated for $\Gamma$ instead of $\mathbb{R}^n$ by the inequality that has been specially proved for this purpose.

The theorem on continuation (theorem on traces) and the related inequality (6), as well as the reverse inequality

$$\|f\|_{H_p^r(\Gamma)} \le c_1 \|f\|_{H_p^{r+1/p}(\mathbb{R}^n)}, \tag{7}$$

were proved in my work 50 years ago. Now, I apply this theorem to the problems of approximation on manifolds. Earlier, this theorem was widely used in the theory of boundary value problems.

Formerly, L. Dirichlet proposed a method for solving a problem, called the Dirichlet problem, of finding on a domain $\Omega$ a harmonic function $U$ with prescribed boundary values $U|_\Gamma = \varphi$, $\Gamma = \partial\Omega$. According to Dirichlet, for the required function $U$, a certain integral attains its minimum; in particular, for $n = 3$,

$$\min_{f|_\Gamma=\varphi} D(f) = \min \iiint\limits_\Omega \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2\right] dx\,dy\,dz = D(U), \qquad U|_\Gamma = \varphi.$$

However, Weierstrass rejected this method and proposed an example of a function $\varphi$, continuous on the boundary $\Gamma$, such that the Dirichlet method is inapplicable since there do not exist any function $f$ with a finite Dirichlet integral whose values on $\Gamma$ are equal to the function $\varphi$ proposed by Weierstrass.

Later, Hilbert showed that the Dirichlet method can be applied if, instead of function $\varphi$ defined on $\Gamma$, one defines on the domain $\Omega$ a function $\Phi$ having a finite Dirichlet integral and seeks a solution to the extremum problem among the functions $f$ that have a finite Dirichlet integral on $\Omega$ and coincide with the function $\Phi$ on $\Gamma$.

However, there still remained a question after Hilbert as to what requirements should a function $\varphi$, defined on $\Gamma$, satisfy in order to be a trace of a function $f$ with a finite Dirichlet integral. Inequalities (6) and (7) give an answer to this question in terms of $H$-classes: it is sufficient that $\varphi \in H_2^r(\Gamma)$, $r > 1/2$.

When proving my theorem on traces, I assumed that the manifold $\Gamma$ is $k$-times continuously differentiable ($\Gamma \in S^k$), where $k > r + \frac{n-m}{p}$.

Later, it was shown that one can assume that $k > r$. This improved estimate was obtained in collaboration with V.P. Il'in [5] as well as with my student V.V. Shan'kov [28] and A. Jonsson [29]. Accordingly, it was assumed in my works that the boundary of the manifold $\Gamma$ is described by continuously differentiable functions. Now, we can generalize this fact; namely, we can assume that the boundary is described by functions satisfying the Lipschitz condition. This results from the following fundamental theorem proved by my disciple O.V. Besov [2]: a function $f$ from the class $B_{p\theta}^r(\Omega)$, where $\Omega \subset \mathbb{R}^n$ is a domain with a Lipschitz boundary, can be continued onto $\mathbb{R}^n$ with the preservation of the class. The continued function satisfies the inequality

$$\|f\|_{B_{p\theta}^r(\mathbb{R}^n)} \le c\|f\|_{B_{p\theta}^r(\Omega)}, \qquad r > 0, \quad 1 \le p \le \infty, \quad 0 < \theta \le \infty,$$

where $c$ is independent of $f$. Here, one should take into account that $B_{p\infty}^r = H_p^r$.

In the one-dimensional case, i.e., when $\Omega$ is a straight-line segment, the relevant theorem was proved by my disciples V.K. Dzyadyk [4] and O.V. Besov [1].

This is the end of the scientific part of this paper.

On April 16, 2000, the Presidium of the Russian Academy of Sciences awarded me the A.N. Kolmogorov Prize for the series of works *Approximation of Functions on Manifolds and Their Continuation*.

I frequently attend two mathematical Schools: those in Voronezh and Saratov. The permanent organizer of the Voronezh school is Professor Yu.V. Pokornyi, Voronezh State University. The Voronezh school held in May 1999 was dedicated to the anniversary of Academician V.A. Sadovnichii, the Rector of the Moscow State University.

I like to visit Voronezh also for sentimental reasons. During the civil war, when I was 13–16, our family lived in the famous Shipov forest, where my father worked as a forest officer, 120 km from Voronezh. I also worked there; I did not attend school but learned from my father. Perhaps, the tragedy of the civil war had a stronger effect on forests than on villages. While attending the Voronezh mathematical school, I always visit the Shipov forest and stand for a while among the trees near a granite block with the scripture: "Here, on July 1921, counter-revolutionary bandits murdered Mikhail Dmitrievich Nikol'skii, an officer of the Shipov experimental forest reserve," my father.

This year, I visited the Saratov mathematical school, which is now headed by Professor Avgust Petrovich Khromov, Saratov State University, after deceased A.A. Privalov, a relative of famous I.I. Privalov.

An essential role in the ideological guidance of this School is played by P.L. Ul'yanov. He also plays an important role in the management of the Voronezh school when it is devoted to the theory of functions.

In March 2000, I managed to visit a mathematical conference in Yekaterinburg, dedicated to the memory of S.B. Stechkin. There is a good mathematical school on the approximation theory in Yekaterinburg, which was founded by Stechkin and is now headed by his student Yu.N. Subbotin, Corresponding Member of the Russian Academy of Sciences.

Here, I also had a chance to soothe my feelings. About 220 km from Yekaterinburg, there is Talitsa—the town where I was born. It is situated to the east of Yekaterinburg along the Siberian highway, 120 km short of Tyumen'. Yurii Nikolaevich Subbotin and Vitalii Ivanovich Berdyshev were so kind that they not only organized my trip to Talitsa in a car but also accompanied me. This was a long trip, and we were pressed for time; therefore, we stayed in Talitsa only few hours and, of course, visited the Forestry school. Last year, it celebrated its centennial. There is a list of first teachers of the Talitsa forestry school in the museum of this school; my father, Nikol'skii is among these teachers.

My father graduated from the Imperial Forestry Institute in St. Petersburg in 1896. After that, he worked two years as an assistant forest officer in Tsarevokokshaisk (presently Ioshkar-Ola), Kazan province. He married there my mother, Lyudmila Mikhailovna Fedorova, who worked as a village teacher that time. In 1899, my father was conferred a title of a silviculturist of the first rank in the St. Petersburg Forestry Institute and was sent to the newly opened Talitsa forestry school as an assistant forester and a lecturer. In Talitsa, my parents gave birth to four children (later, they had six). I am the youngest one; I was born in 1905. In 1906, my family left Talitsa and went to the extreme west of our vast Empire, literally to the border with Germany, where my father had already become a forester (in Avgust forests); now, it is in Poland.

The family chronicle says that we went from Perm' to Moscow by ship along the Kama, Volga, and Oka; I learned to walk on the deck of a ship.

I would like to mention the Mathematical conference in Aktobe (formerly Aktyubinsk) in the fall of 1999. Oleg Vladimirovich Besov and I were suddenly invited there. This is a town in Kazakh steppe, several hundred kilometers from Orenburg far inland Kazakhstan. The organizer of the conference was Professor K.K. Kenzhebaev, the rector of the university in Aktobe. He is a great enthusiast and made a lot for the development of mathematics in Kazakhstan steppes. He was educated in Kiev, being a student of Academician Anatolii Mikhailovich Samoilenko. The latter also visited this conference together with other Ukrainian scientists.

I keep longstanding close relations with Kazakh mathematicians. I should mention my deceased student Tyuleubai Idrissovich Amanov, Corresponding Member of Kazakh Academy of Sciences and the Director of the Institute of Mathematics in Alma-Ata. Presently, I have close relations with another one of my students, Professor Kabdush Nauryzbaev.

In 2000, from May 6 to 21, I visited USA. I was an invited reporter at the International Conference on the Approximation Theory in Nashville, Tennessee. All reporters at the plenary session presented review reports. I decided to present my recent results obtained during the last year; my English is not adequate to make review reports, neither I like to sight-read my papers. All assured that my presentation was good. Before the conference, I could visit the University of Columbia, South Carolina, by invitation of Professor DeVore, a leading scientist in the modern approximation theory. He organized a good research group in his department. The former scientists of the Department of Function Theory from our institute, K.I. Oskolkov and V.N. Temlyakov, play an important role in this group. They both showed great hospitality. In particular, I traveled more than a thousand kilometers over United States with them. There were forests and mainly single-story houses.

In April, this year, another event occurred that was related to my scientific activities in Ukraine. Together with my Ukrainian colleagues N.P. Korneichuk, Academician of the National Academy of Sciences of Ukraine, and A.I. Stepanets, Corresponding Member of the National Academy of Sciences of Ukraine, I was awarded the M.V. Ostrogradskii Prize for the series of works on the theory of approximation of functions.

I arrived at Kiev and attended the General Meeting of the Academy of Sciences of Ukraine to get the diploma. Of course, it was a pleasure; however, I was especially pleased to see that my student from Dnepropetrovsk, Vitalii Pavlovich Motornyi was elected a Corresponding Member of the National Academy of Sciences of Ukraine.

I lived 25 years in Ukraine. Nine years I lived in Chernigov (from 1914 to 1918 and from 1922 to 1925) and 16 years (from 1925 to 1940) in Dnepropetrovsk (formerly Yekaterinoslav). I studied at the Dnepropetrovsk University and was allowed to work there. For one and a half year, I was sent to the Department of Mechanics and Mathematics, Moscow State University, where I defended my Candidate's dissertation; after that, I fulfilled the duties of the Head of the Chair of Function Theory. I became a student of A.N. Kolmogorov, who regularly visited the Dnepropetrovsk University in the 1930s. Under the influence of Kolmogorov, I started to work in the field of approximation of functions. A couple years before the war, Kolmogorov organized a seminar on the function theory at the Department of Mathematics in the Dnepropetrovsk University; in his absence, I was the head of this seminar. Actually, this seminar gave origin to the Dnepropetrovsk school on the approximation theory. During the war, this school was shut down; however, immediately after the war, it was restarted. I have lived (since 1941) in Moscow but, a few years after the war, I regularly visited Dnepropetrovsk to deliver lectures at the university. The following students of mine became outstanding scientists:

A.F. Timan, Professor, Dr. Sci. (Phys.–Math.),

V.K. Dzyadyk, Corresponding Member of the National Academy of Sciences of Ukraine,

N.P. Korneichuk, Academician of the National Academy of Sciences of Ukraine,

V.P. Motornyi, Corresponding Member of the National Academy of Sciences of Ukraine.

After 1940, I did not live in Ukraine; nevertheless, I have maintained close relations with my Ukrainian students. As for Timan and Dzyadyk, I kept close relations with them until their deaths.

Of course, I appreciate that my Ukrainian colleagues remember me. The Ostrogradskii Prize is my Ukrainian award. In 1994, I was awarded the State Prize of Ukraine (together with V.F. Babenko, V.L. Velikin, N.P. Korneichuk, A.A. Ligun, and V.P. Motornyi).

In 1990, the Dnepropetrovsk State University conferred me the title of an Honorary Professor.

The main thing that the Dnepropetrovsk University made for me is the following. In 1925, when I was 20, I decided to enter an institute of higher education. To do this, I had to move to another town. I wanted to be an engineer; therefore, as I came from Chernigov, I applied to the Kiev Polytechnical University. However, they did not accept me, despite my five-year service record and the fact that I was a member of the Komsomol. In those times, institutes of higher education distributed permits (assignments) like permits for sanatoriums. Those who had permits were accepted passing an easy exam. The others were not accepted; they were not offered even a difficult exam. I could not get a permit for a technical college; however, the trade union offered me a permit for the Yekaterinoslav University. Reluctantly, I took this permit and easily entered the Department of Physics and Mathematics of the Yekaterinoslav University. However, I was planning to move to a technical college after a year. Why should I become a mathematician and, hence, a school teacher? To be an engineer is quite a different thing!

However, having studied a year at the Department of Mathematics of the Dnepropetrovsk University, I realized that my vocation is to become a professional mathematician, no matter how much money I would earn. I abandoned my thoughts about entering a technical college once and for all. I am grateful to the Dnepropetrovsk University for this.

On May 3, this year, the Institute of Mathematics organized a one-day conference dedicated to my 95th birthday. There arrived my friends: Polish Academicians Czeslaw Olech and Zbigniew Ciesielski, a Bulgarian Academician Blagovest Sendov, my student from Dnepropetrovsk V.P. Motornyi, Corresponding Member of the National Academy of Sciences of Ukraine, and V.D. Stepanov, Corresponding Member of the Russian Academy of Sciences from Khabarovsk.

In the presidium of the session, there were Academician Yu.S. Osipov, President of the Russian Academy of Sciences and the Director of the Steklov Institute of Mathematics; Academician V.A. Sadovnichii, the Rector of the Moscow State University; Academician A.A. Samarskii; Academician of the Polish Academy of Sciences Ch. Olech; Professor V.M. Filippov, Minister of Education of the Russian Federation; I.I. Mel'nikov, Doctor of Pedagogics, Member of the State Duma; and my student L.D. Kudryavtsev, Corresponding Member of the Russian Academy of Sciences.

There were many colleagues of mine from our Institute, from the Academy of Sciences, the members of our seminar, etc.

Papers were read, among which there was a telegram from V.V. Putin, read by Yu.S. Osipov. The telegram ended with the words "Let me congratulate you with all my heart on account of the oncoming Day of Great Victory." There also were a diploma of the Kolmogorov Prize from the Presidium of the Russian Academy of Sciences, an address from the State Duma signed by G.N. Seleznev, a personal letter from Minister V.M. Filippov with the words "It is a special honor to me to be a student of your school," and an address of the Rector of the Moscow State University V.A. Sadovnichii, who mentioned my ties with the Moscow State University. It was solemn and cheerful.

It is a great honor to be awarded the celebrated Kolmogorov Prize, especially because I was one of his students; apparently, the oldest of the living ones. When I was proposed as a candidate for this prize, the Academic Council of the Institute voted unanimously with only one abstention (myself).

There was a high-level refreshment. It is hard to imagine what a good time we had.

Here I do not touch upon my activities in the Institute. I have a manuscript in my bookcase on this matter.

However, the celebrations did not yet end. When I came back from America, the Moscow Institute of Physics and Technology (MIPT) organized a large banquet.

At this banquet, there arrived respectable graduates of MIPT who occupy various posts in and outside the MIPT, from a rector, prorectors, and deans, to professors, assistant professors, laboratory assistants, and secretaries.

On the honorary place beside me, there were Professor N.N. Kudryavtsev, the Rector of the MIPT; the former rectors Academician O.M. Belotserkovskii and N.V. Karlov, Corresponding Member of the RAS; G.N. Yakovlev, the Head of the Department of Mathematics and a disciple of our seminar, Corresponding Member of the Russian Academy of Pedagogical Sciences; and his predecessor in the Department, my student L.D. Kudryavtsev, Corresponding Member of the RAS. There were greetings, recollections; and there was something to recollect.

In the first years of this famous Institute, I delivered lectures together with P.L. Kapitsa, L.D. Landau, I.G. Petrovskii, B.N. Delone, S.L. Sobolev, and A.A. Dorodnitsyn. During a certain period, I was the Head of the Department of Higher Mathematics and, permanently, a professor in this department. I wrote *A Course of Mathematical Analysis* for the students of MIPT and for physicists and mathematicians in general.

I delivered lectures in the MIPT during 50 years without interruptions. First, I gave lectures on certain subjects of analysis and later on the analysis proper. I admit that I did not expect such a friendly touch from the MIPT.

Formally, I have already been awarded by the MIPT quite recently, when I was 90. The Moscow Institute of Physics and Technology made me its first Honorary Professor and an Honorary Master. Having been conferred the title of an Honorary Master of the MIPT, I became an honorary engineer.

When I was a small boy, my father used to say "Our Serega is a mathematician; he will be an engineer." My activities in the MIPT are described in [30].

I have another thing I am keen on. This is working at school. To be more exact, this is the teaching of mathematics at school. I think, I cannot call it a hobby, because it is more than hobby.

It is a long time since I have realized that, in order to put into practice one's point of view concerning the teaching of mathematics at school, one has to write his own textbooks of mathematics. Now, I decided to do this.

The textbooks of arithmetic and algebra with a full set of problems are published by a lithographic technique (Research Institute of the Content and Methods of Teaching, Academy of Pedagogical Sciences of the USSR). By an administrative decree of M.A. Prokof'ev, the USSR Minister of Education, the Prosveshchenie publishers issued 50 thousand copies of the experimental textbook (in three volumes) *Algebra 6, 7, 8* by S.M. Nikol'skii, M.K. Potapov, and N.N. Reshetnikov (1984, 1985, and 1986). These textbooks were experimentally tested in certain regions of Dnepropetrovsk and Krivoi Rog.

During 1999–2000, a new period started in this activity—the publication of approved textbooks for normal schools.

The following textbooks were approved by the Ministry of Education of Russia and published: *Arithmetic 5, 6* and *Algebra 7, 8, 9.* A preliminary textbook *Algebra 10* is published (on the money of Moscow State University). (The authors: S.M. Nikol'skii, M.K. Potapov, N.N. Reshetnikov, and A.V. Shevkin.)

When I was 90, the MIPT journalists asked me how it occurred that I have lived so long. "What did you eat since your childhood to become so smart?" they asked in Ukraine.

As for my food, I still eat everything: anything fried, stewed, and sweet. Earlier, I smoked. However, when I got glaucoma, I gave up smoking. I drink only to keep company. Earlier, sometimes I used to overstep the limits; but now I know when to stop. If I will get glaucoma in my other eye, I think I will give up drinking at all.

Our famous long-living mathematicians P.S. Aleksandrov and A.N. Kolmogorov did not drink vodka; they drank wine, and drank it in limited amounts. Boris Nikolaevich Delone did not drink any wine. I outlived their age limits.

Of course, I used to go for a walk a lot; I walked in forests and mountains, swam, and rowed a lot; Delone walked in the mountains more than I did. As for water, Andrei Nikolaevich and Pavel Sergeevich swam, and swam a lot, in water of any temperature (sometimes alongside floating blocks of ice). Kolmogorov used to ski in frosty winter days in just shorts. This was impossible for me. It was enough to stay five minutes in cold water (not even ice-cold water) or five minutes in March air without clothes, even in sunny weather, to get a burst of polyarthritis (a kind of rheumatism), and I would be confined to bed. Boris Nikolaevich was also an absolutely hardened man. When he was 80, I witnessed how he refreshed himself in a mountain river flowing out from just under shelf ice. This was in a mountain camp in Frunze.

Another classical example is Academician Ivan Matveevich Vinogradov, an extremely strong man from birth. Once, they loaded a grand piano on him, and he carried it alone to the third floor. He liked wrestling, short walks, and swimming in sea in a seaside resort (Batiliman). It turns out that I have also outlived this strong man, who lived a plain and comfortable life.

I have not yet outlived the age limit (96 years) of Dmitrii Evgen'evich Men'shov. The life of this famous mathematician passed in unpretentious conditions. It was exclusively devoted to science. He lived a plain life and every day used to make long walks and regularly played tennis.

I used to make my walking exercises one day a week, except for vacation periods. On ordinary working days I usually abandoned my exercises. There were certain short periods when I went in for morning exercises. I just could never come to enjoying jogging; when you are jogging, all the way you think when it will come to an end. I like walking more. I liked walking for a long time, with only one stop to make a campfire, cook kasha, make tea, and talk to friends.

Now, there remain only reduced forest walks and working on a vegetable garden (digging, weeding, and watering) of all this.

I think I made it clear why I answered the MIPT journalists "I don't know, I wonder myself; evidently, only God knows."

Finally, I note that all five aforementioned mathematicians persistently worked on mathematics up to the end of their long lives. I mean that mathematics is by no means harmful to a human. Answers to the questions posed should be sought for in a different area.

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